An Introduction to Quantum Field Theory

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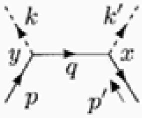
ingoing for initial-state particles and outgoing for final-state particles. This follows immediately from the expansions of ψ and $\bar{\psi}$, where the annihilation operators $a_{\mathbf{p}}$ and $b_{\mathbf{p}}$ both multiply $e^{-ip\cdot x}$ and the creation operators $a_{\mathbf{p}}^{\dagger}$ and $b_{\mathbf{p}}^{\dagger}$ both multiply $e^{+ip\cdot x}$. On internal fermion lines (propagators), the momentum must be assigned in the direction of particle-number flow (for electrons, this is the direction of negative charge flow). This requirement is most easily seen by working out an example from first principles. Consider the annihilation of a fermion and an antifermion into two bosons:



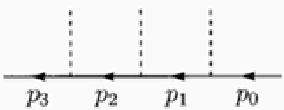
$$= \langle \mathbf{k},\mathbf{k}'|\int d^4x\, \phi\bar{\psi}\psi \int d^4y\, \phi\bar{\psi}\psi\, |\mathbf{p},\mathbf{p}'\rangle$$

$$\sim \int d^4x \int d^4y\, \bar{v}(p')e^{-ip'\cdot x} \int \frac{d^4q}{(2\pi)^4}\, \frac{i(\not{q}+m)}{q^2-m^2} e^{-iq\cdot(x-y)}\, u(p)e^{-ip\cdot y}.$$

The integrals over x and y give delta functions that force q to flow from y to x, as shown. On internal boson lines the direction of the momentum is irrelevant and may be chosen for convenience, since $D_F(x-y) = D_F(y-x)$.

It is conventional to draw arrows on fermion lines, as shown, to represent the direction of particle-number flow. The momentum assigned to a fermion propagator then flows in the direction of this arrow. For external antiparticles, however, the momentum flows opposite to the arrow; it helps to show this explicitly by drawing a second arrow next to the line.

Third, note that in our examples the Dirac indices contract together along the fermion lines. This will also happen in more complicated diagrams:



$$\sim \bar{u}(p_3)\cdot \frac{i(\not{p}_2+m)}{p_2^2-m^2}\cdot \frac{i(\not{p}_1+m)}{p_1^2-m^2}\cdot u(p_0). \tag{4.117}$$

Finally, let's take a moment to worry about fermion minus signs. Return to the example of the fermion-fermion scattering process. We adopt a sign convention for the initial and final states:

$$|\mathbf{p},\mathbf{k}\rangle \sim a_{\mathbf{p}}^{\dagger}a_{\mathbf{k}}^{\dagger}|0\rangle, \qquad \langle \mathbf{p}',\mathbf{k}'| \sim \langle 0|\, a_{\mathbf{k}'}a_{\mathbf{p}'}, \tag{4.118}$$

so that $(|p,k\rangle)^{\dagger} = \langle p,k|$. Then the contraction

$$\langle \mathbf{p}',\mathbf{k}'|(\bar{\psi}\psi)_x\,(\bar{\psi}\psi)_y|\mathbf{p},\mathbf{k}\rangle \sim \langle 0|\, a_{\mathbf{k}'}a_{\mathbf{p}'}\;\bar{\psi}_x\psi_x\;\bar{\psi}_y\psi_y\;a_{\mathbf{p}}^{\dagger}a_{\mathbf{k}}^{\dagger}|0\rangle$$

can be untangled by moving $\bar{\psi}_y$ two spaces to the left, and so picks up a factor of $(-1)^2 = +1$. But note that in the contraction

$$\langle \mathbf{p}',\mathbf{k}'|(\bar{\psi}\psi)_x\,(\bar{\psi}\psi)_y|\mathbf{p},\mathbf{k}\rangle \sim \langle 0|\, a_{\mathbf{k}'}a_{\mathbf{p}'}\;\bar{\psi}_x\psi_x\;\bar{\psi}_y\psi_y\;a_{\mathbf{p}}^{\dagger}a_{\mathbf{k}}^{\dagger}|0\rangle\,,$$