# Calculus

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The simplest numbers are the "counting numbers":

$$1, 2, 3, \ldots$$

The fundamental significance of this collection of numbers is emphasized by its symbol **N** (for natural numbers). A brief glance at PI-PI 2 will show that our basic properties of "numbers" do not apply to N — for example, P2 and P3 do not make sense for **N**. From this point of view the system N has many deficiencies. Nevertheless, **N** is sufficiently important to deserve several comments before we consider larger collections of numbers.

The most basic property of **N** is the principle of "mathematical induction." Suppose $P(x)$ means that the property $P$ holds for the number $x$. Then the principle of mathematical induction states that $P(x)$ is true for all natural numbers $x$ provided that:

(1) $P(1)$ is true.

(2) Whenever $P(k)$ is true, $P(h + 1)$ is true.

Note that condition (2) merely asserts the truth of $P(k + 1)$ under the assumption that $P(k)$ is true; this suffices to ensure the truth of $P(x)$ for all x, if condition (1) also holds. In fact, if $P(1)$ is true, then it follows that $P(2)$ is true (by using (2) in the special case $k = 1$). Now, since $P(2)$ is true it follows that $P(3)$ is true (using (2) in the special case $k = 2$). It is clear that each number will eventually be reached by a series of steps of this sort, so that $P(k)$ is true for all numbers $k$.